RESEARCH PAPERS ON KNOWLEDGE, INNOVATION AND ENTERPRISE
VOLUME 13, ISSUE 1, JULY 2025, PP. 83-105

Kerr & Schwarzschild Black Hole Photon Trajectories – Models & Simulations

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Received 21.03. 2025; received in revised form 14. 07. 2025; accepted 21.07. 2025

Abstract

In this research, spinning (Kerr) and a non-spinning (Schwarzschild) black hole simulation models are introduced. Via the simulation models, the photon trajectories are computed by solving the geodesics equations. The black holes are simulated via equatorial trajectories based on the spherical symmetry of the solutions. The developed models aid in researching the surfaces and geodesics of black holes under various (customizable) conditions (via the input parameters of the simulation models). A rotating black hole, such as a Kerr black hole, has angular momentum, causing it to drag spacetime around it. A non-rotating black hole, such as a Schwarzschild black hole, does not spin and hence has a perfectly spherical event horizon. Based on the research I conducted on Black Holes, I further developed a (hypothetical) theory that focuses on the potential interconnections among the Schwarzschild radius, the Plank length, and the Compton wavelength at the Plank scale. The established theory is focused on providing a different conversation angle when it comes to discussions on General Relativity and quantum mechanics.

Keywords: Black Holes, Kerr, Schwarzschild, Geodesics, Light Trajectory, Simulation Models, Compton Wavelength, Plank Scale, Quantum Mechanics, General Relativity

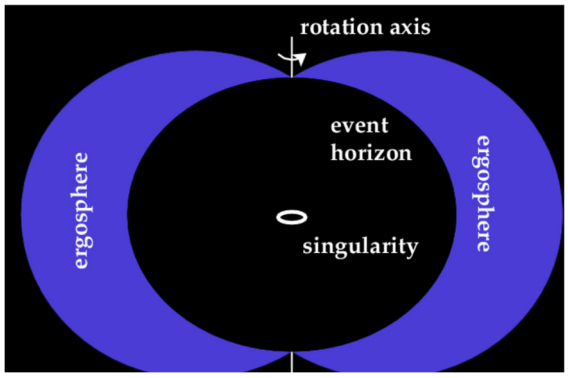
Introduction to Black Holes and the Ergosphere

In astrophysics, the ergosphere depicts a region that is located outside a rotating black hole's outer event horizon (see Figure 1). It is labeled that way as it is theoretically possible to extract energy and mass from this region. The ergosphere touches the event horizon at the poles of a rotating black hole and extends to a greater radius at the equator. A black hole with modest angular momentum has an ergosphere with a shape approximated by an oblate spheroid, while faster spins produce an even more pumpkin-shaped ergosphere. The equatorial (maximal) radius of an ergosphere is the Schwarzschild radius, the radius of a non-rotating black hole. The polar

(minimal) radius is also the polar (minimal) radius of the event horizon that can be as little as half the Schwarzschild radius for a maximally rotating black hole.

In general, as a black hole rotates (theoretically some do not), it twists spacetime in the direction of the rotation at a speed that decreases with distance from the event horizon. This process is known as either the Lense–Thirring effect or frame-dragging. Because of this dragging effect, an object within the ergosphere cannot appear stationary in respect to an outside observer (at a distance) unless that object would be moving faster than the speed of light. The speed necessary for such an object to appear stationary decreases at points further out from the event horizon, until at some distance the required speed is negligible. The set of all such points defines the ergosphere surface, labeled as the ergosurface. The outer surface of the ergosphere is identified as the static surface or static limit. All this is due to the fact that world lines change from being time-like outside the static limit to being space-like inside of it. It is the speed of light that impulsively defines the ergosphere surface. Such a surface would appear as an oblate that is concurrent with the event horizon at the pole of rotation, but at a greater distance from the event horizon at the equator. Outside this surface, space is still dragged, but at a lesser rate. A suspended plumb, held stationary outside the ergosphere, will experience an infinite/diverging radial pull as it approaches the static limit.

Figure 1: Black Hole System – Kerr Black Hole



At some point it will start to fall, resulting in a gravitomagnetically induced spinward motion. An implication of this dragging of space is the existence of negative energies within the ergosphere. Since the ergosphere is outside the event horizon, it is still possible for objects that enter that region with sufficient velocity to escape from the gravitational pull of the black hole. An object can gain energy by entering the black hole's rotation and then escaping from it, thus taking some of the black hole's energy with it (making the maneuver similar to the exploitation of the Oberth effect

around normal space objects). The Oberth effect, a principle in rocket science, states that a rocket engine is more efficient at increasing kinetic energy (and therefore orbital energy) when operating at higher speeds, implying that a burn at periapsis (closest point to the body being orbited) yields a greater increase in velocity than a burn at apoapsis (farthest point).

This process of removing energy from a rotating black hole was proposed by the mathematician Roger Penrose and is known as the Penrose process (Penrose, 1965). The maximal amount of energy gain possible for a single particle via this process is 20.7% (in terms of its mass equivalence), and if this process is repeated by the same mass, the theoretical maximal energy gain approaches 29% of its original mass-energy equivalent. As this energy is removed, the black hole loses angular momentum, and thus the limit of zero rotation is approached as spacetime dragging is reduced. In the limit, the ergosphere no longer exists. This process is considered a possible explanation for a source of energy of such energetic phenomena as gamma-ray bursts.

As discussed by Williams (Williams, 2002), results obtained from computer models show that the Penrose process is capable of producing the high-energy particles that are being emitted from quasars and other active galactic nuclei. The size of the ergosphere, the distance between the ergosurface and the event horizon, is not necessarily proportional to the radius of the event horizon, but rather to the black hole's gravity and its angular momentum. A point at the poles does not move, and thus has no angular momentum, while at the equator a point would have its greatest angular momentum. This variation of angular momentum that extends from the poles to the equator is what gives the ergosphere its oblate shape. As the mass of the black hole or its rotation speed increases, the size of the ergosphere increases as well.

A Deeper Dive into Black Holes

Based on Einstein's theory of General Relativity, many researchers believe that every black hole contains a singularity, a point of infinite density and gravity where the laws of physics as we know break down. Theoretical black holes like Kerr black holes do have singularities. In this research, the labels *theoretical* and *realistic* black holes refer to the state of knowledge in 2025 where several configurations of black holes are being discussed (see below). We do know that black holes are real, the actual setup though is what researchers still are trying to nail down. Hence, one thing that scientists are interested in is whether black holes that are formed based on realistic conditions (such as a stellar collapse) have singularities. Penrose showed that inside event horizons (realistic or not), the affine length of a light ray is finite. This affine length is a way of generalizing the *concept of length* for light, because the true spacetime length of any light ray is always zero. From this, Penrose argues that if the ray has a finite affine length, it

must terminate in a singularity. Thus, all black holes, even realistic ones, should have singularities.

Using his metric, Kerr showed the existence of light rays with finite affine length that never terminate. So, Penrose's argument is technically invalid. Kerr only has to find a single light ray of finite affine length inside a black hole that does not terminate in order to render Penrose's argument invalid. But Kerr does not prove the non-existence of singularities, he is merely disqualifying the argument that they are required to exist. Black holes only have three physical properties. So, all we need to describe them is:

- How massive they are
- How fast they are rotating (if at all)
- How much charge they carry (if any)

Now, all black holes have some amount of mass. It is not really possible to turn that property off. We can increase or decrease the mass, but they will always be going to have mass. The other two properties though *can* be turned off. We can easily cancel the charge of an entity. If the entity represents an object that is two fundamental charges too positive, we can add two electrons and now, the entity is uncharged.

Likewise, if an object is spinning too fast, we can have it interact with entities that are spinning in the opposite direction. The equations of physics teach us that angular momentum has to be conserved. So, if an entity is spinning, it keeps spinning until we do something about it. By adding particles that spin the opposite direction of a black hole, we decrease its rate of rotation. Well, that leaves us with four distinct black hole configurations that are labeled after the scientists that provided the equations describing these objects:

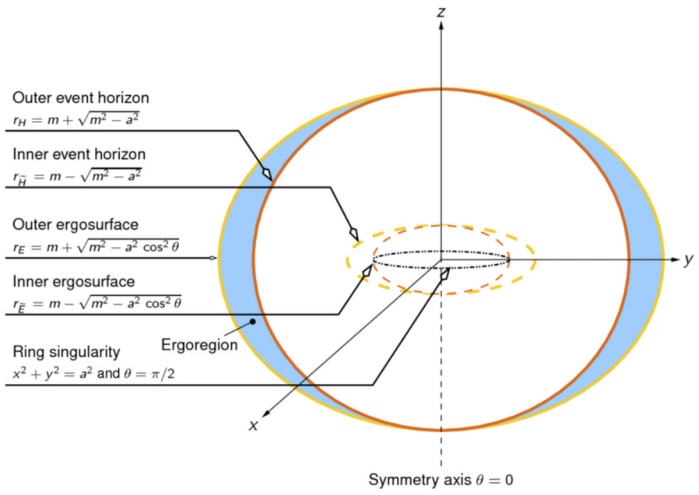
1. No Charge, No Rotation - Schwarzschild Black Hole
2. No Charge, Rotation - Kerr Black Hole
3. Charge, No Rotation - Reissner-Nordström Black Hole
4. Charge, Rotation - Kerr-Newman Black Hole

As a consequence of General Relativity, rotating black holes drag the surrounding space with them. This is the effect known as frame dragging. The effect changes and shapes the event horizon (the point beyond which nothing can escape) and creates a new region known as the ergosphere (see Figure 1 and 2). The event horizon depicts a perfect sphere for a non-rotating black hole. For a rotating black hole, it gets slightly flattened at the poles and wider at the equator (so shaped similar to our planet).

The outer boundary of the ergosphere is also a flattened sphere, but larger than the event horizon. Its inner boundary is the event horizon. The ergosphere is the region between these two surfaces, as shown in Figure 2. As already discussed, in this region, it is *theoretically* possible to extract

energy from the black hole and use it for other purposes. Even outside the ergosphere, the frame dragging effect takes place. Essentially space itself begins to spin *with* the black hole. If we were near it, the parts of us closest to the black hole would feel a force pulling us along with the rotation, while the parts furthest from us would feel as though they were being pulled in the opposite direction. In reality, all of us is being pulled *with* the rotation, but at different rates. Assuming one hand closer, that hand is being pulled faster than the hand farther away, while our torso is somewhere in between, it feels as though *both* are being pulled away.

Figure 2: Kerr Solution for a Rotating Black Hole



Another major difference between rotating and non-rotating black holes deals again with the singularity. In general terms, the singularity is normally described as the dead center of a black hole. For a non-rotating one, the singularity basically depicts a single point. For a rotating black hole though, something interesting happens. For a Kerr black hole, the singularity depicts a ring. The ring exists because *in classical mechanics* a single point cannot have angular momentum by its own. In quantum mechanics though, it is perfectly fine for a single point to have angular momentum (spin). But, as of now, physicists have not yet been able to unify gravity and quantum mechanics. As the question of singularities deals with both, we have to label that problem as *unsettled* in 2025.

The singularity forms when matter is compressed so tightly that no other force of nature can balance it. In a normal star, like the Sun, the inward pull of gravity is balanced by the outward pressure of the nuclear reactions in its core. In collapsed stars known as white dwarfs or neutron stars, other forces prevent the ultimate collapse. If there is too much mass

in a given volume though, the object reaches a critical density where nothing can prevent its ultimate collapse, forming a black hole.

As gravity overcomes the other forces of nature, a singularity follows its own strange rules of physics. Time and space as we know are crushed out of existence, and gravity becomes infinitely strong. As the distance from the singularity increases, the escape velocity decreases. The escape velocity can be described as the speed that an object hast to travel in order to get away. For Earth, the escape velocity is approximately seven miles per second. In other words, a spacecraft has to travel at least that fast to escape Earth's gravitational pull. At a certain distance from the singularity, the escape velocity equals to the speed of light (approximately 186,000 miles/second). This distance is known as the Schwarzschild radius (honoring Karl Schwarzschild who defined it). The actual Schwarzschild radius depends on the mass of the black hole. For a black hole as massive as the Sun, the radius is approximately two miles. For every extra solar mass, the radius increases by two miles.

This radius enfolds the singularity in a zone of blackness. In other words, that is why a black hole is black. It provides the black hole with a visible surface known as the event horizon. The horizon does not represent a solid surface though. It simply depicts the *point of no return* for anything that approaches the black hole. Once any object, either a vessel or a light particle crosses inside the horizon, it cannot escape. It is trapped inside the black hole. If the black hole does not rotate, its gravitational influence on its environment is rather straightforward. If the black hole is spinning though, then its gravitational effects are more complicated. To reiterate, in that scenario it actually pulls the fabric of spacetime along with it (frame dragging in the ergosphere). It has to be pointed out that technically, a black hole does not have an equator as it represents a singularity or a point source. The event horizon radius (Schwarzschild radius) can have an equator based on its direction of angular momentum or the observers' point of view as they approach a non-spinning black hole. Non-rotating black holes do have one photon sphere, while spinning black holes have two that merge. They also have a photon circle, an area or region of space where gravity is so strong that photons are forced to travel in orbits. These spheres can also have an equator.

Do we Really Understand Singularity?

I have to point out that some researchers such as Carlo Rovelli and Hal Haggard (Rovelli, 2024) have a different point of view when it comes to black holes and singularity. They depict a black hole as a funnel (nothing extraordinary here). But the devil is in the details. They state that it may be natural to think that strange things happen at the bottom of the funnel, down in the center of the black hole. But that is not the case. In the center of the funnel, there is only the falling star, so they argue that we are not in singularity territory there.

To illustrate, let us picture a very old black hole, down there in the funnel, at the bottom, time has slowed. Outside, millions of years may have passed, while down there just a few fractions of a second have ticked away. The star is still falling at the bottom of the long funnel that is stretching and narrowing because in its timeframe, no more than fractions of a second have passed. So, the zone where the distortions become infinite, where Einstein's equations do not work anymore, the really interesting zone is not there. So where is it? They argue that it is in the future. As the diameter of the funnel narrows, the cylinder becomes more curved, like a tube that is rolled tighter. The more the funnel is narrowed, the greater the distortion of spacetime. When it reaches the Planck scale, the scale where we expect space and time to be seriously affected by quantum phenomena, we enter the region where these marvels imply that Einstein's equations no longer apply. If we ignore this argument, space has collapsed, and time ends here.

So, the argument by Rovelli and others is that the region of the singularity, the quantum region, is in the future. It is where the tube is squeezed into a line and becomes infinitely long. It is not the center of the back hole, again, there is only a falling star at the center. To better understand what happens in a black hole, we should not think of it as a stationary cone with the singularity at its center. We have to visualize it as a long tube with the star that generated it at the bottom! The tube gets longer and thinner and in the future is squeezed into a line. So, the singularity is not at the center, it is after in time! After studying Rovelli's research on black holes and white holes, I have to acknowledge that his explanations make the most sense to me.

A white hole is the same as a black hole, just reversed in time. To make that work, spacetime has to quantum tunnel from a black hole, beyond the singularity, to enter the realm of a white hole. That quantum tunneling effect of spacetime does not happen in space and time. It is a phenomena that is neither special nor temporal, it is an instantaneous quantum transition from one configuration of space to another. In 1974, Stephen Hawking' theoretical discovery that black holes emit heat, represents another quantum tunneling effect. Hawking was the first scientist to actually compute the temperature of a black hole (Hawking, 1974). In general, radiated heat carries away energy. As it loses energy, the black hole gradually loses mass (mass is energy) and so becomes lighter and lighter. That all implies that its horizon shrinks, an effect that is known to scientists these days as an evaporating black hole.

Schwarzschild Black Hole Simulations – Photon Trajectories

This section of the research discusses the mathematical equations used to develop our non-rotating black hole simulation model. In this study, the geodesics equation (Equation 1) is solved by using the Schwarzschild met-

ric. A geodesic represents the shortest path, or *straightest line*, between two points on a curved surface or in a curved space, generalizing the concept of a straight line in Euclidean geometry. Hence, geodesics represent the curved space equivalent to straight lines in flat space. They signify the locally shortest path between two points on a curved surface or in a curved space.

On a sphere, geodesics are arcs of great circles (such as the equator or a meridian) while in general relativity, geodesics are the paths followed by objects that are moving freely (that are not subject to external forces) through spacetime. From a mathematical perspective, geodesics are identified by a differential equation that describes the curves that minimize distance on a given surface or in a given space. The concept of geodesic deviation illustrates how nearby geodesics diverge or converge, which is important while studying the effects of gravity and tidal forces.

$$\frac{d^2\gamma^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu}\frac{d\gamma^\mu}{dt}\frac{d\gamma^\nu}{dt} = 0$$

Equation 1

The symmetry in our problem statement implies that only equatorial trajectories are required ($\theta = \varpi/2$). Based on that, a regular differential equation describing the photon trajectory can be derived (Equation 2).

$$\frac{d^2u}{d\phi^2} - \frac{3R_s}{2}u^2 + u = 0$$

Equation 2

By stating that $u(\phi) = \frac{1}{r(\phi)}$ and $R_s = \frac{2MG}{c^2}$ we can stipulate that R_s depicts the black hole radius, M references the black hole mass, G represents the gravitational constant, while c portrays the speed of light in a vacuum. An implicit Runge-Kutta method is used in this study, as this method is often the preferred choice for solving a stiff differential equation, as it can handle the rapid changes in the solution without becoming unstable. A stiff differential equation includes components that change rapidly compared to others, leading to a large disparity in timescales within the system.

On the other hand, explicit Runge-Kutta methods can rather easily fail while dealing with stiff systems that require very small step sizes to maintain accuracy. In essence, the implicit nature allows the method to *look ahead* at the solution for the next (time) step, providing greater stability in the presence of large variations in the derivative. Ergo, the chosen method for this study allows to remain accurate, whereas an explicit method would diverge. Prior to integrating the equation, our second order equation has to be transformed into two first order equations. Therefore, Equation 2 is converted into Equation 3:

$$v_0 = u[1]$$
$$v_1 = \frac{3R_s}{2}u[0]^2 - u[0]$$

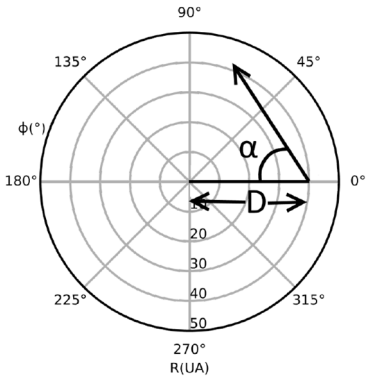
Equation 3

In Equation 3, $u[0] = u(\varphi)$ while $v_0(0)$ represents the initial position. Further, $u[1] = \frac{du}{d\phi}$ while $v_1(0)$ describes the initial direction. A backwards ray-tracing method is implemented to determine the origin of a photon. Backward ray tracing describes a ray tracing technique that starts at the eye and traces rays towards the light source. So, this describes the opposite direction that photons normally travel. Ergo, the initial conditions (see Figure 2) can be described as:

$$v_0(0) = \frac{1}{D}$$
$$v_1(0) = \frac{1}{D.tan(\alpha)}$$

Equation 4 & 5

Figure 2: Initial Conditions (Polar Coordinate System) - Black Hole in the Center



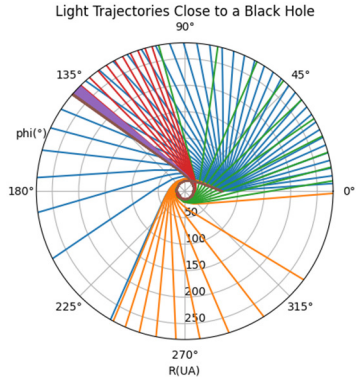
D depicts the distance from the black hole while α describes the angle describing the initial direction.

The obtained results can be used to either visualize trajectories or simply return the *deviated_angle* that corresponds to the actual pixel position obtained for a *seen_angle*. Various trajectories are visualized in Figure 3. In this study, the angles are identified and quantified via Equations 6 and 7.

$$seen_angle = \pi - \alpha$$
$$deviated_angle = \phi + asin(\frac{D}{R}\sin(\phi))$$

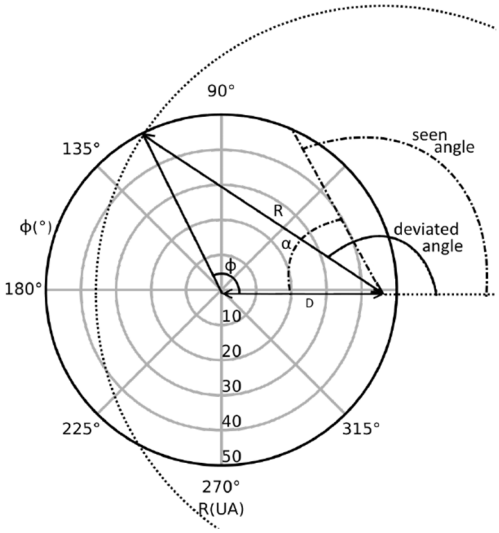
Equations 6 & 7

Figure 3: Light Trajectories Close to a Black Hole.



Modeled trajectories for distance D=70 and vision radius R_s=10. The various colors depict the different precision steps that are utilized to cover all possible alpha scenarios.

Figure 4: Relation Among Seen and Deviated Angles and α, φ.



D reflects the distance from the black hole, R describes the vision radius, α is the initial angle while φ defines the final angle.

The actual angle relations are portrayed in Figure 4 while Equations 8 and 9 (see below) provide the mathematical abstractions embedded into the established simulation models.

$$\psi = \pi - \phi - asin(\frac{D}{R}\sin\phi)$$
$$deviated_angle = \pi - \psi$$

Equations 8 & 9

Our simulation model is developed for equirectangular images so, (x, y) coordinates correspond to (φ, θ) coordinates. To further explain this, an equirectangular image depicts a flat image that represents a 360-degree view of a spherical scene (such as a world map). In our scenario, it is necessary to apply the (φ_2, θ_2) deviated pixel colors onto the (φ, θ) seen pixels. The spherical coordinates (φ, θ) are transformed into Cartesian coordinates to compute β and rotate the pixels on the equator using a rotation matrix (see Equation 10).

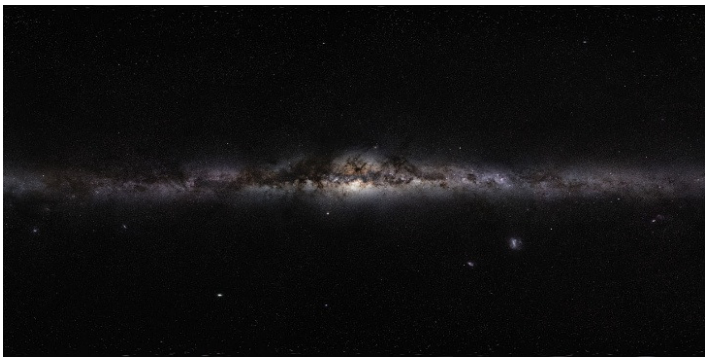
$$\beta = -atan(\frac{z}{y})$$

Equation 10

The new coordinates provide; $\varphi' = seen_angle$ and $\theta' = \varpi/2$. The interpolation is utilized to identify the $deviated_angle = \varphi'_2$. In our solution, the interpolation is executed with $\varphi' \in [0, \varpi]$. So, while $\varphi' > \varpi$, φ' converts to $2\varpi - \varphi'$ as φ'_2 adapts $2\varpi - \varphi'_2$ (all this only works for Schwarzschild black holes). Next, φ'_2 is transformed into Cartesian coordinates and rotated back via an angle ($\alpha - \beta$). Ultimately, the Cartesian coordinates are transformed into spherical coordinates in order to specify that (φ_2, θ_2) = (x_2, y_2). When the calculations are made for pixels inside the black hole, this function returns (-1, -1) and the corresponding pixels are skipped. In our simulation model, loop structures browse every pixel and map the (x_2, y_2) pixel color (RGB) information to the (x, y) pixel (see Figure 7). An offset function is used to simulate a moving black hole or observer.

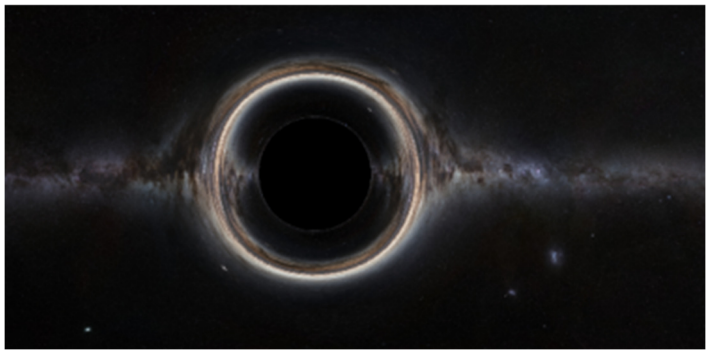
For the Schwarzschild and the Kerr simulation models, the initial background is an image of the Milkey Way panorama as seen in Figure 6. The models, based on the chosen parameters, map a block hole onto the background image while the simulations are running. Figure 7 depicts the output generated by the Schwarzschild simulator. For all the simulations, actual movies are generated to study the black hole light trajectory behaviors.

Figure 6: Milkey Way Panorama Image used as the Background



Base image (curtesy of eso.org)

Figure 7: View of a Schwarzschild Black Hole from a Distance



Our Simulation Model: Configuration: R_s=10, D=70. UA with a 90 degrees image

Kerr Black Hole Simulations – Geodesics

To restate, black holes are described as regions in spacetime that exhibit gravitational acceleration scenarios that are so strong that nothing can escape. Black holes were predicted by Einstein's theory of General Relativity and are now being studied by researchers around the world. The center of our Galaxy is where we have a huge black hole, so not being at the center of our Galaxy is actually a good thing.

Black holes hold three independent physical properties based on the no-hair theorem, (1) mass, (2) angular momentum, and (3) charge. Uncharged, rotating black holes are defined via the Kerr metric and so are labeled as the Kerr black holes. It is common to express the Kerr metric in Boyer-Lindquist coordinates:

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr}{\Sigma} \sin^2\theta \, dt \, d\phi + \frac{(r^2+a^2)^2 - a^2 \Delta \sin^2\theta}{\Sigma} \sin^2\theta \, d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma \, d\theta^2$$

Equation 11

where r, φ, and θ represent a standard spherical coordinate system that can be expressed as:

$$\Delta(r) = r^2 - 2Mr + a^2 \quad und \quad \Sigma(r,\theta) = r^2 + a^2 \cos^2\theta$$

Equation 12

In Equation 12, Σ is referred to as the singularity of the black hole. It can be derived via Σ=0. Everything is in geometric units G=c=1 and M=1. Based on that, the simulation model allows visualizing the surfaces of a rotating black hole (see Figure 8).

The Kerr parameter a can be adjusted to investigate various scenarios such as the Schwarzschild metric (a=0) or an extreme Kerr solution (a=1). The surfaces are calculated as stated in the following few formulas. The inner and the outer event horizon is depicted in Equation 13 while inner and outer ergosphere is outlined in Equation 14:

$$r_H^{\pm} = M \pm \sqrt{M^2 - a^2}$$

Equation 13

$$r_E^{\pm} = M \pm \sqrt{M^2 - a^2 \cos\theta^2}$$

Equation 14

The singularity Σ is given by r = 0 ∧ θ = π/2. The simulation models allow visualizing the surfaces in two different coordinate systems, either in Cartesian coordinates or the Boyer-Lindquist coordinates. As discussed, the geodesics depict the straightest-possible lines as stipulated by the geometry of spacetime. Their motion is described by the geodesic equation depicted here:

$$\frac{d^2x^{\kappa}}{dt^2} + \Gamma^{\kappa}_{\mu\nu} \frac{dx^{\mu}}{dt} \frac{dx^{\nu}}{dt} = 0$$

Equation 15

where $\Gamma^{\kappa}_{\mu\nu}$ reference the Christoffel symbols of the metric. The geodesics in Kerr spacetimes are calculated via the formulas In Equation 16. In Equation 16, κ = Q + L2 + a2(E2+μ). The elements μ, E, L, and Q depict the constants of motion. In the simulation models, the initial conditions are given by (r, θ, φ, τ, pr, pθ). The models utilize the 4th order Runge-Kutta method to integrate the system of ordinary differential equations. Via the simulation model, it is possible to adjust the step size (the integration can be paused).

$$\dot{r} = \frac{\Delta}{\Sigma} p_r$$

$$\dot{\theta} = \frac{p_\theta}{\Sigma}$$

$$\dot{\varphi} = \frac{2arE + (\Sigma - 2r)L/\sin^2\theta}{\Sigma\Delta}$$

$$\dot{\tau} = E + \frac{2r(r^2+a^2)E - 2arL}{\Sigma\Delta}$$

$$\dot{p}_r = \frac{1}{\Sigma\Delta}\left(((r^2+a^2)\mu - \kappa)(r-1) + r\Delta\mu + 2r(r^2+a^2)E^2 - 2aEL\right) - \frac{2p_r^2(r-1)}{\Sigma}$$

$$\dot{p}_\theta = \frac{\sin\theta\cos\theta}{\Sigma}\left(\frac{L^2}{\sin^4\theta} - a^2(E^2+\mu)\right)$$

Equation 16

Kerr Black Hole – Light Trajectory

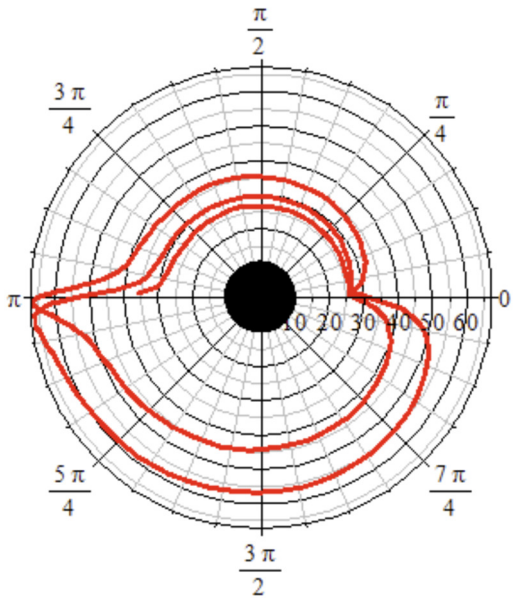
A light ray traveling near a Kerr black hole follows a complex, spiraling trajectory due to the black hole's rotation, potentially orbiting multiple times around the event horizon before ultimately falling in. This happens with an exact path that depends on the initial direction and angle of the light ray relative to the black hole's spin axis. This phenomenon is described by the Kerr metric in General Relativity that shows that light can even become *dragged* by the black hole's rotation within the ergosphere, the region just outside the event horizon where all objects must co-rotate with the black hole (see Figure 8).

Depending on the light ray's path, an observer far away from the black hole may see multiple images of the same source due to the light bending around the black hole multiple times. Within the ergosphere, light rays are forced to rotate in the same direction as the black hole, regardless of their initial trajectory. Light rays that orbit in the same direction as the black hole's rotation (prograde) can have significantly different trajectories compared to those orbiting in the opposite direction (retrograde). In the case of an extremely rotating Kerr black hole, light rays traveling in the prograde direction may not experience a singularity in their trajectory.

To summarize the mathematics implemented in the simulation models, to describe the light trajectory around a Kerr black hole, we used the Kerr metric as expressed in the Boyer-Lindquist coordinates (r, θ, φ), where the key formula for a light ray is $ds^2 = 0$ and the relevant parameters are the black hole mass (M) and the angular momentum (J) represented by the spin parameter a (a = J/M). The major components of the Kerr metric for light ray calculations are described in Equation 17 (line element where $\rho^2 = r^2 + a^2\cos^2\theta$):

$ds^2 = - (dt^2 - a^2\sin^2\theta\, d\varphi^2/\rho^2) + \rho^2/ (dr^2 + d\theta^2)$ Equation 17

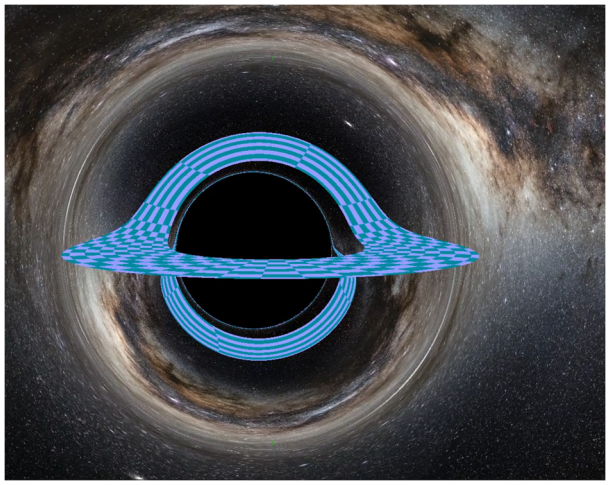
Figure 8: Kerr Black Hole Light Trajectory



Kerr Black Hole Simulation Results and Conclusions

Studies conducted via our simulation models on light trajectories involving a Kerr black hole revealed that due to the angular momentum, light can be significantly deflected and even orbit around the black hole in complex patterns. This may create multiple images of a source depending on the viewing angle, with the most prominent feature being the existence of a region called the ergosphere where even light must co-rotate with the black hole's spin. This effect is particularly pronounced for rapidly rotating black holes. To reiterate, while analyzing the light trajectory around a Kerr black hole via the models, the major revelation was that the light path is significantly affected by the black hole's spin (see Figure 9).

Figure 9: Kerr Black Hole Light Trajectory Simulation



Photon Trajectories obtained via the Kerr simulation modes

The angular rotation causes it to follow a twisted, non-planar trajectory, with the possibility of prograde (same direction as the spin) or retrograde (opposite direction) orbits depending on the initial light ray direction. This all may potentially exhibit multiple reflections near the photon sphere (due to the frame-dragging effect), that results in a complex and distorted shadow when observed from far away.

Black Hole Properties

Based on our current understanding, most if not all *realistic* black holes are likely to be rotating to some degree as of the natural processes that create them (such as the accumulation of matter from a companion star). Nonetheless, as in this study, the concepts of non-rotating black holes is frequently used (in theoretical calculations) to simplify the mathematical aspects of the projects. But an interesting question remains and that is if Kerr black holes even exist? Personally, I would answer the question with *maybe* and *maybe not*. A Kerr black hole is technically an entity that has angular momentum but has no charge.

I would argue though that most black holes are both spinning *and* have charge. I base that statement on the fact that there are just way too many charged particles in space. Hence, it would be very difficult to perfectly neutralize the charge of a black hole. Some no-charge black holes may occasionally pop into existence. But I would state that they would only last for a very short time, basically only when just the right amount of charge has been absorbed. In theory though, they would surely absorb another electron or positron just as quickly and so would again have

a charge. Having said that, in my opinion, it would be even harder to stop a black hole from spinning. Having outlined all of this, my money is on *realistic* rotating, charged black holes. But, with the vast Cosmos we deal with, a perfect Kerr black hole may be flickering in and out of existence once in a blue moon. As quantum mechanics teaches us, any scenario, no matter how unlikely, has a greater than zero probability to pop up somewhere.

Final Remarks and Future Research Directions

Special Relativity has shown that the notion of the present is subjective. But could there be a connection between heat and time? I would say so. There is a difference between the past and the future, only if there is heat involved. Heat is linked to probability. Probability is linked to the fact that our interactions with the world do not register the fine details or reality. The flow of time emerges from physics, but it is not an exact description of things as they are. Time emerges in the context of statistics and thermodynamics.

So, I do agree when scientists describe the enigma of *time*, as the *present* does not exist in an objective sense any more than *here* exists objectively, this all is just a temporal phenomenon. I would like to point out that I strongly believe though that time sits at the center of the tangles of problems raised by the interaction of gravity, quantum mechanics, and thermodynamics. Let us be clear, gravity and quantum mechanics do not play well together in 2025.

Well, Steven Hawking, using quantum mechanics, demonstrated that black holes are always *hot*, they emit heat (Hawking, 1974). The heat of a black hole depicts a quantum effect upon an object, a black hole, that is gravitational in nature! It is the individual quanta of space, the vibrating molecules that heat the surface of black holes and generate the heat. Hence, this phenomena involves all three sides of the problem, quantum mechanics, General Relativity (gravity), and thermal science. Therefore, I believe that this could be the smoking gun that someday, maybe, will allow us to combine quantum, gravity, and thermodynamics.

Having stated that, I always imagined that the more we study black holes, the better we understand black holes, the better our chances to further research how to combine quantum mechanics and gravity. While working on this research project, I developed my own thoughts and ideas on the subject and the following few paragraphs depict the state where I am at right now. The following theory is just that, it is a theory, a model that I developed to push the discussions on quantum gravity along.

Black Holes—Quantum Mechanics and General Relativity at the Planck Scale

The objective of this research is to explore and discuss a plausible, and in my opinion potentially profound relationship among the Planck length, the Schwarzschild radius, and the Compton wavelength at the Planck scale. In a nutshell, the Schwarzschild radius depicts the radius of a sphere surrounding an object where the escape velocity equals to the speed of light. The Plank units are in my opinion extremely significant as at that level, we cannot disregard either gravity or quantum mechanics, as both effects are equally important. At that level, it is necessary to develop a quantum gravity theory to describe the Universe. Further, below the Plank time, time just does not exist and that in itself represents another interesting factor that we do have to consider in many physics related studies.

Table 1: Plank Units

- Planck Length: $\ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616\,229(38) \times 10^{-35}$ m
- Planck Time: $t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}} = 5.391\,247(60) \times 10^{-44}$ s
- Planck Mass: $M_P = \sqrt{\frac{\hbar c}{8\pi G}} \approx 4.341 \times 10^{-9}$ kg $= 2.435 \times 10^{18}$ GeV/c^2
- Planck Charge: $q_P = \sqrt{4\pi\epsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}} \approx 1.875\,5459 \times 10^{-18}$ coulombs

The focus is on highlighting the (hypothetical) convergence possibility and to provide some intuition on how to potentially quantify quantum gravity. The theory is based on how a black hole's Schwarzschild radius, derived from its mass, becomes comparable to the Planck length when the mass is equivalent to the Planck mass. The study further incorporates the Compton wavelength of particles, in principle photons that despite having no rest mass do have energy. The argument made is that the interrelation among the three elements suggests that at the Planck scale, conventional boundaries between quantum mechanics and gravity get rather fuzzy, indicating a potentially strong connection among matter, energy, and spacetime.

The possible convergence of these notions at the Planck scale may lead to further discussions on the state of our current understanding of physics in general. The prospective blending of quantum mechanics and general relativity at that scale proposes in my opinion that spacetime may not behave in the classical sense that we are used to and that both, quantum mechanics and general relativity are equally significant at that level. Fur-

ther, this theory may actually offer additional insights into phenomena's such as black hole thermodynamics and the nature of singularities.

It is paramount to distinguish between rest mass (invariant mass) and energy-based mass, especially in the context of quantum mechanics and general relativity. The rest mass depicts the mass of an object measured at rest relative to the observer and so portrays a fundamental property of particles. In contrast, energy-based mass refers to the notion that mass can be derived from energy via Einstein's equation $E = mc^2$ (Equation 18). In high-energy physics scenarios, mainly for massless particles such as photons, their effective mass can be deduced from their energy ($E = hc/\lambda$ – Equation 19). Therefore, while the rest mass remains constant, the energy-based mass can vary based on the particle's energy, leading to distinct implications in regard to gravitational and quantum related circumstances. In this study, we define the Planck length as ℓ_P, the Schwarzschild radius R_s of a black hole with energy E equal to the Compton wavelength λ of a photon hc/λ. Hence, the relationship can be expressed as:

$\ell_P = R_s = \lambda$ (Equation 20)

In other words, the statement made is that the energy E of a black hole with Schwarzschild radius R_s is equal to the energy of a photon hc/λ with a Compton wavelength λ. The assertion made here suggests that we can potentially link three concepts, (1) the Planck length, (2) the Schwarzschild radius, and (3) the Compton wavelength at the Planck scale.

To advance the discussion, the theory provides the following rationalizations. The Schwarzschild radius R_s of a black hole is determined by its mass. At extremely small (mass) scales, principally when the mass is equivalent to the Planck mass, the Schwarzschild radius befalls akin to the Planck length ℓ_P. Ergo, a black hole with a mass equal to the Planck mass M_P would have an event horizon radius that is approximately equal to the Planck length. In mathematical terms this can be expressed as:

$R_s = 2G \cdot M_P/c^2 \approx \ell P$ (Equation 21)

At this fundamental length, quantum gravitational effects are expected to become profound, and hence the argument made is that general relativity and quantum mechanics both play essential roles at that level. The Compton wavelength of a particle (with a rest mass indicated by m) is inversely correlated to the mass of the particle. In other words, as the mass increases, its Compton wavelength decreases. For photons that do have no rest mass ($m = 0$), the Compton wavelength is established via their energy. In this scenario, the Compton wavelength is characterized as $\lambda = h/mc$, which simplifies to $\lambda = h/E$ while considering photons. Here, h depicts the Plank constant, m the mass of the particle in question, while c portrays the speed of light in a vacuum. For a photon, the energy associated with the Compton wavelength can so be expressed as:

$E = hc/\lambda$ (Equation 22)

This association demonstrates that the energy of a photon is inversely proportional to its Compton wavelength. As the wavelength in-

creases, the energy decreases, emphasizing the fundamental connection between the wavelength and the energy in the context of quantum mechanics. If we associate the photon energy with the energy of a black hole (the rest energy of a black hole with Planck mass M_P), the wavelength of the photon is now directly comparable to the Planck length. The Planck mass describes the minimum mass of a classical object M_o that corresponds to its Schwarzschild radius R_s. The Planck mass M_p is approximately 21.8 micrograms (μg) and can be quantified via a rather simple equation that is comprised of the speed of light c, the reduced Planck constant $\hbar$, as well as the gravitational constant G. The principal statement made by the theory discussed here is that at the Planck scale, a photon with the same energy as the rest mass energy of a Planck mass black hole will have a wavelength that is equal to the Schwarzschild radius of that black hole. In mathematical terms, this implies that:

$\ell_P = R_s = \lambda$ (Equation 23)

Hence, this hypothesis suggests that (1) the photon's wavelength and (2) the black hole's event horizon are equal in size at the quantum level. So, the energy of the photon corresponds to the energy required to form a black hole with a radius that is equal to the Planck length. This indicates that at the Planck scale, there is a potentially profound link between quantum mechanics and gravity. The Schwarzschild radius (a classical gravitational concept) and the Compton wavelength (a quantum mechanical perception) are equal at this scale. If this theory holds, the proclamation made is that the conventional disconnect (discussed and elaborated on by so many other studies) between quantum mechanics and general relativity may get really fuzzy under the described circumstances.

The Planck mass represents the smallest (probable) mass for a black hole to even exist. The energy of a photon with a Compton wavelength equal to the Planck length is gigantic. The assertion made here is that this photon behaves just like a black hole. Any photon with such a small wavelength (the Planck length) contains so much energy that it can be described as a black hole with a mass that equals to the Planck mass. All this basically depicts a fundamental quantum gravitational effect in the sense that light (that is conventionally massless) may possibly (under extreme circumstances) exhibit a black hole alike behavior. Hence, there may be a profound, theoretically noteworthy association here. So, at the Planck scale, where quantum mechanics and general relativity merge, a black hole's Schwarzschild radius, the energy of a photon, and its Compton wavelength all possibly converge! This may indicate that in the realm of quantum gravity, the particularities among matter, energy, and spacetime are actually deeply interlinked.

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References

Anastassi Z., Simos, T., (2005). An optimized Runge–Kutta method for the solution of orbital problems, *Journal of Computational and Applied Mathematics*, vol. 175, no. 1, pp. 1–9.

Autti S., et al., (2020). Fundamental dissipation due to bound fermions in the zero-temperature limit, *Nature Communications*, Vol.11, number 4742.

Barden, J., Kerr, M. (1970). Black Holes, *Nature*, Vol. 226, Issue 5240, pp. 64-65.

Chandrasekhar, S. (2020). The Mathematical Theory of Black Holes, Clarendon Press, Oxford Classic Texts in the Physical Sciences.

Draper, T., Benjamin, K., Ripken, C. & Saueressig, F. (2020). Finite Quantum Gravity Amplitudes: No Strings Attached, Phys. Rev. Lett. 125, 181301 – (Published 27 October 2020).

Gelles, Z., Prather, B., et al., (2021). The role of adaptive ray tracing in analyzing black hole structure, *The Astrophysical Journal*, Vol. 912, no. 1, p. 39.

Hamilton, A. (2010). "Journey into and through a Reissner-Nordström black hole", University of Colorado, USA.

Hawking, S. W. (1974). Black Hole Explosions? *Nature News*.

Hawking, S. W. (1966). Properties of expanding universes. Doi.org.

Hawking, S. W. (2021). Radiation and the Sonic Black Hole - Technion.

Kerr, R. (2023). Do Black Holes have Singularities? 10.13140/ RG.2.2.34286.38723.

Konstantinov, S., "Epistemological Optimism of Knowledge of the Physics of the Universe", Open Access *Journal of Physics* Volume 3, Issue 1, (2019).

Mizner Ch., Thorne K., & Wheeler J. (1977). Gravity Volume 3, Publishing House.

Paganini, C., Ruba, B., & Oancea, A. (2016). Characterization of null geodesics on Kerr spacetimes, arXiv preprint arXiv:1611.06927.

Penrose, R. (1965). Gravitational Collapse and Space-Time Singularities, Phys. Rev. Lett. 14, 57.

Puget, J. (2014). The sight on the relic background, Scientific American, No.9.

Rovelli, C. (2023). White Holes, Riverhead Books.

Shikin, V. (2013). Low – frequency anomalies of effective mass of charged clusters in liquid helium, Low Temperature Physics, 39(10), p.837. DOI: 10.1063/1.4825362.

Tommaso, T. (2020). Hubble detects smallest known dark matter clumps, by ESA/Hubble Information Centre, Home Astronomy & Space, Jan 9.

Viergutz, C. (1993). Image generation in Kerr geometry. i. analytical investigations on the stationary emitter-observer problem, Astronomy and Astrophysics, Vol. 272, p. 355.

Williams, R. (1991). Extracting X-Rays Gamma Rays, and Electron-Positron Pairs from Supermassive Kerr Black Holes Using the Penrose Mechanism, Harvard University.

Williams, R. (2002). The Gravitomagnetic Field and Penrose Processes, AstroPH.